e. Find an equation for $f(g(x))$ explicitly in terms of x. Enter this equation as $f_4(x)$ and plot it on the same screen as the other three functions. What similarities and what differences do you see for $f_4(x)$ and $f_3(x)$?

11. *Square and Square Root Functions:* Let f and g be defined by

$f(x) = x^2$, where x is any real number

$g(x) = \sqrt{x}$, where the values of x make $g(x)$ a real number

a. Find $f(g(3))$, $f(g(7))$, $g(f(5))$, and $g(f(8))$. What do you notice in each case? Make a conjecture: "For all values of x, $f(g(x)) =$ __?__ and $g(f(x)) =$ __?__."

b. Test your conjecture by finding $f(g(-9))$ and $g(f(-9))$. Does your conjecture hold for negative values of x?

c. Plot $f(x)$, $g(x)$, and $f(g(x))$ on the same screen. Use approximately equal scales on both axes, as in Figure 1-4n. Explain why $f(g(x)) = x$, but only for nonnegative values of x.

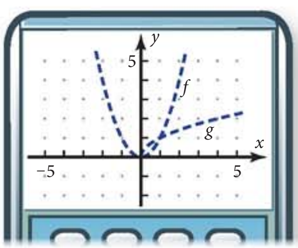


Figure 1-4n

d. Deactivate $f(g(x))$, and plot $f(x)$, $g(x)$, and $g(f(x))$ on the same screen. Sketch the result.

e. Explain why $g(f(x)) = x$ for nonnegative values of x, but $g(f(x)) = -x$ (the opposite of x) for negative values of x. What other familiar function has this property?

12. *Horizontal Translation and Dilation Problem:* Let f, g, and h be defined by

$f(x) = x^2 \qquad -2 \le x \le 2$

$g(x) = x - 3 \qquad$ for all real values of x

$h(x) = \frac{1}{2}x \qquad$ for all real values of x

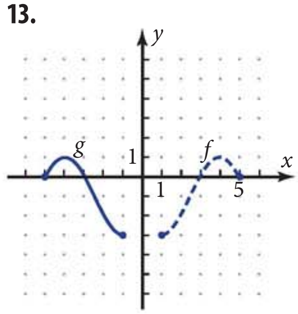
a. $f(g(x)) = f(x - 3)$. What transformation is applied to function f by composing it with g?

b. $f(h(x)) = f\left(\frac{1}{2}x\right)$. What transformation is applied to function f by composing it with h?

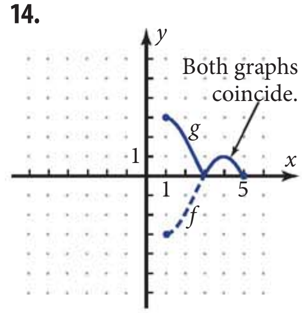
c. Plot the graphs of f, $f \circ g$, and $f \circ h$. Sketch the results. Do the graphs confirm your conclusions in parts a and b?

For Problems 13 and 14, find what transformation will turn the dashed graph (f) into the solid graph (g).

13.



14.



15. *Linear Function and Its Inverse Problem:* Let f and g be defined by

$f(x) = \frac{2}{3}x - 2 \qquad g(x) = 1.5x + 3$

a. Find $f(g(6))$, $f(g(-15))$, $g(f(10))$, and $g(f(-8))$. What do you notice in each case?

b. Plot the graphs of f, g, $f \circ g$, and $g \circ f$ on the same screen. How are the graphs of $f \circ g$ and $g \circ f$ related? How are the graphs of $f \circ g$ and $g \circ f$ related to their "parent" graphs, f and g?

c. Show that $f(g(x))$ and $g(f(x))$ both equal x.

d. Functions f and g in this problem are said to be *inverses* of each other. Whatever f does to x, g undoes. Let $h(x) = 5x - 7$. Find an equation for the inverse function of h.